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ENERGY DISTRIBUTION OF A SURFACE SOURCE IN AN INHOMOGENEOUS HALF-SPACE*

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Relations describing the energy flux in an elastic medium in terms of the surface loads and Green matrix components are used to obtain formulas suitable for determining the density of the total amount of energy transported by various type waves across a horizontal plane, side surface of a cylinder, and surface of a sphere of large radius. This makes possible the complete determination of the energetic balance in a vertical inhomogeneous half-space.

When the power from an oscillatory source is transmitted through the earth, it is important to know the distribution of energy between the elastic, various type waves. The known /1, 2/ relations connecting the energies of the longitudinal, transverse and Rayleigh waves have been obtained jor a model of a homogeneous elastic half-space. The inhomogeneity of the earth's core however stipulates the redistribution of the energy between the various type waves and in different directions. The same problem is encountered in designing antivibration coatings from composites, multilayer constructions, acoustoelectronic devices on surface waves, etc. The present paper deals with the development of the method of its solution.

1. We consider an elastic or viscoelastic inhomogeneous half-space $(-\infty \leqslant x,\ y \leqslant \infty,\ -\infty \leqslant z \leqslant 0)$ with depth-dependent properties $\lambda = \lambda(z),\quad \mu = \mu(z),\ \rho = \rho(z)$. Here ρ is density, $\lambda = \lambda_1 + i\lambda_2$, $\mu=\mu_1+i\mu_2$ are the Lamé coefficients of the medium and we have $\lambda_2 \leqslant 0,\ \mu_2 \leqslant 0;\ \lambda_2 = \mu_2 = 0$ for the elastic medium. The steady state oscillations of the medium $\mathbf{v} = \mathrm{Re}\,[\mathbf{u}e^{-i\omega t}]$ are generated by harmonic surface loads $\boldsymbol{\tau} = \mathrm{Re}\,[\mathbf{q}e^{-i\omega t}],\ (x, y) \in \Omega$ defined in some region Ω. Outside Ω the surface of the medium is load-free; $\mathbf{u}(x, y, z), \mathbf{q}(x, y)$ are the complex amplitudes of the displacements and surface stresses. In the course of investigating the steady state oscillations it is expedient to use as the measure of change of energy within a volume, its change averaged over the oscillation period $T = 2\pi/\omega$ /3/

$$E = \frac{1}{T}\int_0^T \frac{\partial E_s}{\partial t}\,dt = \iint_S \rho_E\,dS \tag{1.1}$$

$$\rho_E = -\omega/2\,\mathrm{Im}\,(\mathbf{u}\cdot\boldsymbol{\sigma}),\qquad \boldsymbol{\sigma} = \lambda\mathbf{n}\,\mathrm{div}\,\mathbf{u} + 2\mu\,\frac{\partial\mathbf{u}}{\partial n} + \mu\,(\mathbf{u}\times\mathrm{rot}'\mathbf{u})$$

Here E_s is the energy within a certain volume of the medium bounded by the surface S, ρ_E is the energy flux density and σ is the complex amplutide of the stress vector, the stresses appearing on the area element with outward normal $\mathbf{n}$ to the surface S.

The displacements $\mathbf{u}$ of the medium can be expressed in terms of the surface stresses $\mathbf{q}$ /4/

$$\mathbf{u}(x, y, z) = \frac{1}{4\pi^2}\int_{\Gamma_1}\int_{\Gamma_2} K(\alpha_1, \alpha_2, z)\,\mathbf{Q}(\alpha_1, \alpha_2)\,e^{-i(\alpha_1 x+\alpha_2 y)}\,d\alpha_1\,d\alpha_2 \tag{1.2}$$

$$\mathbf{Q}(\alpha_1, \alpha_2) = \{Q_1, Q_2, Q_3\} = \iint_\Omega \mathbf{q}(x, y)\,e^{i(\alpha_1 x+\alpha_2 y)}\,dx\,dy$$

$$K(\alpha_1, \alpha_2, z) = \left\|\begin{matrix} -i(\alpha_1^2 M + \alpha_2^2 N)/\alpha^2 & -i\alpha_1\alpha_2(M-N)/\alpha^2 & -i\alpha_1 P \\ -i\alpha_1\alpha_2(M-N)/\alpha^2 & -i(\alpha_1^2 N + \alpha_2^2 M)/\alpha^2 & -i\alpha_2 P \\ \alpha_1 S/\alpha^2 & \alpha_2 S/\alpha^2 & R \end{matrix}\right\|$$

The quantities M, N, P, R, S are functions of $\alpha = \sqrt{\alpha_1^2 + \alpha_2^2}, z$ and are determined from the boundary value problems for the systems of differential equations with variable coefficients in case of continuous and inhomogeneous media or with piece-wise continuous in the case of multilayered media. The elements of the matrix K are functions regular in α and z, have a finite number of real poles in variable α and an enumerable number of complex poles. The

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integration contours Γ_1, Γ_2 which coincide almost everywhere with the real axis and deviate from it only when going around the real poles of the integrand functions, are chosen in accordance with the principle of limit absorption. The choice of the contours Γ_1, Γ_2 and the properties of K were discussed thoroughly in /4/.

The main difficulty encountered while using the representation (1.2) consists of the fact that when the properties of the medium depend on the depth in an arbitrary manner, then the elements of the matrix K cannot be written in an explicit form. Methods for their construction based on numerical solutions of the corresponding boundary value problems were given earlier in detail in (*). The stability of these methods is ensured by the preliminary separation of the exponential and strongly oscillatory terms of the solution in explicit form.

2. Let us denote by S the surface $z = \text{const}$ parallel to the surface of the medium and situated at the depth z. From (1.1) and (1.2), using the Parseval equality and making the substitution

$$\alpha_1 = \alpha \cos\gamma,\ \alpha_2 = \alpha \sin\gamma;\ 0 \leqslant \gamma \leqslant 2\pi,\ \alpha \in \Gamma$$

we obtain

$$E = \frac{\omega}{2}\frac{1}{4\pi^2}\operatorname{Im}\int\limits_\Gamma G(\alpha, z)\,\alpha\,d\alpha \tag{2.1}$$

$$G(\alpha, z) = \int\limits_0^{2\pi} (\mathbf{\Sigma}\cdot\mathbf{U})\,d\gamma =$$

$$F_1[\mu(M' + S)M^* + ((\lambda+2\mu)S'/\alpha^2 - \lambda M)S^*] + F_2\mu N'N^* +$$
$$F_3[\alpha^2\mu(R + P')P^* + ((\lambda+2\mu)R' - \alpha^2\lambda P)R^*] +$$
$$F_4[\mu(M' + S)P^* + ((\lambda+2\mu)S'/\alpha^2 - \lambda M)R^*] +$$
$$F_5[\mu(R + P')M^* + ((\lambda+2\mu)R'/\alpha^2 - \lambda P)S^*]$$

$$F_1 = \int\limits_0^{2\pi} Q_{12}Q_{12}^*\,d\gamma,\quad F_2 = \int\limits_0^{2\pi} Q_{21}Q_{21}^*\,d\gamma$$

$$F_3 = \int\limits_0^{2\pi} Q_3Q_3^*\,d\gamma,\quad F_4 = \int\limits_0^{2\pi} Q_{12}Q_3^*\,d\gamma,\quad F_5 = \int\limits_0^{2\pi} Q_3Q_{12}^*\,d\gamma$$

$$Q_{12} = (\alpha_1Q_1 + \alpha_2Q_2)/\alpha^2,\ Q_{21} = (\alpha_2Q_1 - \alpha_1Q_2)/\alpha^2$$

Here $\mathbf{U}, \mathbf{\Sigma}$ is the Fourier transform of $\mathbf{u}, \boldsymbol{\sigma}$. The asterisk denotes complex conjugates, the conjugate functions have the conjugate arguments $\alpha_1^*, \alpha_2^*, \alpha^*$ and a prime denotes the derivatives in z. The function $G(\alpha, z)$ is complex valued in some bounded domain of variation of α and has double poles ξ_k on the real axis, distributed to the right of the zone of complex values. Therefore we can split E into two components

$$E = E_V + E_R \tag{2.2}$$

$$E_V = \frac{\omega}{2}\frac{1}{4\pi^2}\operatorname{Im}\int\limits_0^{\varkappa} G(\alpha, z)\,\alpha\,d\alpha,\quad E_R = \frac{\omega}{2}\frac{1}{4\pi}\sum_k \operatorname{res} G(\alpha, z)\alpha|_{\alpha=\xi_k}$$

Here $\varkappa$ is the upper boundary of the zone of complex values; for the homogeneous half-space we have $\varkappa = \rho\omega^2/\mu$, while for a layer of finite thickness we have $\varkappa = 0$ and $E_V \equiv 0$. Below we show that E_V is the energy of the volume waves passing across the plane $z = \text{const}$, and E_R is the energy transported across this plane by the Rayleigh type waves.

Let us turn our attention to the choice of the contour Γ, using the principle of limit absorption /5/. We assume that the internal friction in the medium θ is different from zero. In this case the poles ξ_k of the elements of the matrix K situated on the positive part of the real axis when $\theta = 0$, become complex and are displaced into the upper half-plane. One of the poles ξ_k can also be displaced into the lower half-plane. This occurs in the case of irregular poles associated with the corresponding "inverse" waves (see /4/). The integrand function $G(\alpha, z)\,\alpha$ contains in the neighborhood of some pole ξ_k the terms

$$c_k/(\alpha - \xi_k),\ d_k/(\alpha - \xi_k^*);\ c_k,\ d_k = \text{const}$$

If $\theta \neq 0$, then $G(\alpha, z)$ has no real singularities and the contour Γ coincides with the real axis. Let θ tend to zero by deforming the contour Γ at the points of emergence of ξ_k onto

*) Glushkov E.V. and Gluskova N.V., Calculation of the energy of elastic waves generated by surface sources in a stratified half-space. Rostov-on-Don, Dep. v VINITI, No.5827-81, 1981.

the real axis in such a manner, that the pole does not intersect the contour. The contour Γ can be deformed since $G(\alpha, z)$ is an analytic function. The terms of the integrand function of the form $c_k/(\alpha - \xi_k)$ are bypassed by the contour Γ from below for the regular poles, and the terms $d_k/(\alpha - \xi_k^*)$ from above (ξ_k^* is displaced downwards) for the irregular poles the procedure is reversed. Consequently we have

$$E_R = \frac{\omega}{2}\frac{1}{4\pi}\sum_k j_k(c_k - d_k) \tag{2.3}$$

($j_k = 1$ for the regular and $j_k = -1$ for the irregular poles). Thus by Γ we understand a set of contours, every one of which is chosen separately for different terms of $G(\alpha, z)\,\alpha$.

In determining c_k and d_k the danger arises of referring the functions of the form $(f^*)^* = f$ to the nonconjugated functions. In expanding these functions into the Laurent series, different representations are obtained for the complex ξ, e.g.

$$\alpha = \xi + (\alpha - \xi),\ (\alpha^*)^* = \xi^* + (\alpha - \xi^*)$$

(when $\theta \neq 0$, ξ and ξ^* are displaced in opposite directions). Therefore the rule $(f^*)^* = f$ cannot be used in the derivation of (2.3) and the all asterisks must be retained to the end. The representation (2.2) makes it possible to determine the amount of energy of the volume and Rayleigh waves transmitted from the surface source to the medium, to obtain its distribution between the different volumes of the medium contained e.g. between the planes $z = z_1$ and $z = z_2$ or between the different layers of a multilayer medium, to follow the effect of the properties of the inhomogeneous medium on the dependence on z of the amount of energy passing across the plane $z = \mathrm{const}$, etc.

Numerical computations were carried out with all physical quantities reduced to the dimensionless form. The surface stresses $\mathbf{q}$ and Lamé coefficients λ, μ were referred to the some characteristic value of the shear modulus of the medium μ_0, the density ρ to the characteristic density of the medium ρ_0 and the linear quantities to the characteristic linear dimension a. In this cases the generalized frequency $\overline{\omega} = \omega a\sqrt{\rho_0}/\sqrt{\mu_0}$ is used as the frequency and the forces are given in $\mu_0 a^2$. The energy flux per period $T = 2\pi/\overline{\omega}$ is obtained in terms of the units $E_0 = a^2\mu_0^{3/2}\rho_0^{-1/2}$ and in what follows the bar above ω will be omitted.

Fig.1 depicts the dependence of the energy frequency entering the medium from the normal (subscript z) and tangential (subscript x) load distributed uniformly over a circle of unit radius; $E_{V,z}$, $E_{V,x}$ and $E_{R,z}$, $E_{R,x}$ denote the parts of the energy taken up by the volume and the Rayleigh waves respectively. The medium is an elastic, two-layer half-space of thickness $h = 4$. The parameters of the upper layer (medium 1) and lower half-space (medium 2) are $\lambda_1 = 0.08$, $\mu_1 = 0.08$, $\rho_1 = 0.5$ and $\lambda_2 = 10$, $\mu_2 = 1$, $\rho_2 = 1$ respectively, and this corresponds to the following ratio of the rates of propagation of the longitudinal $v_{p,i}$ and transverse $v_{s,i}$ $(i = 1, 2)$ waves:

$$\frac{v_{s,1}}{v_{p,1}} = \frac{1}{\sqrt{3}},\quad \frac{v_{s,2}}{v_{p,2}} = \frac{1}{2\sqrt{3}},\quad \frac{v_{p,1}}{v_{p,2}} = \frac{1}{5}$$

The dashed lines in Fig.1 denote the energy entering a homogeneous half-space, with the properties of the upper layer.

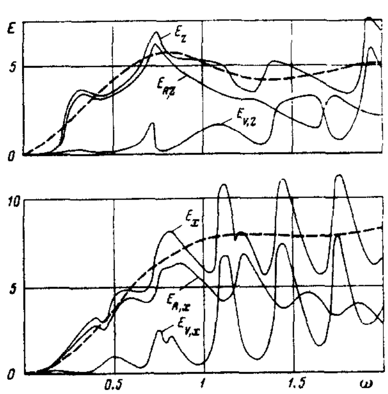

Fig.1

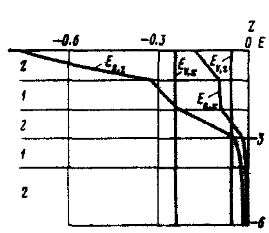

Fig.2

Fig.2 depicts the dependence of the energy of the volume and Rayleigh waves passing across the plane $z = \mathrm{const}$ in a five-layer half-space, on the depth z. The top layers are of thickness $h_i = 1$ $(i = 1, \ldots, 4)$ and the properties of the layers alternate (medium 2-medium 1) $\omega = 0.5$. We see that the energy of the Rayleigh waves $E_{R,z}$, $E_{R,x}$ diminishes with z since there is an energy leakage in the horizontal direction, while the energy of the volume waves remains constant.

3. Let S denote the side surface of a cylinder of radius $r = \sqrt{x^2 + y^2} \gg 1$ contained between the planes $z_1, z_2 = \mathrm{const}$

$$E_R = \int_{z_1}^{z_2}\int_0^{2\pi} \rho_R r\, d\varphi\, dz;\quad x = r\cos\varphi,\ y = r\sin\varphi,\ 0 \leqslant \varphi \leqslant 2\pi \tag{3.1}$$

Here ρ_R denotes the energy flux density across the side surface of the cylinder. The displacements $\mathbf{u}$ have the following asymptotic representation (*):

$$\mathbf{u}(x, y, z) = \sum_k \mathbf{a}_k e^{i\zeta_k r}/\sqrt{r} + O(r^{-3/2}), \quad r \to \infty \tag{3.2}$$

$$\mathbf{a}_k(x, y, z) = \sqrt{\frac{i\zeta_k}{2\pi}} \operatorname{res} K(-\alpha\cos\varphi, -\alpha\sin\varphi, \alpha, z)|_{\alpha=\zeta_k} \cdot \mathbf{B}_k$$

where for the axisymmetric function $\mathbf{q}(x, y)$ we have

$$\mathbf{B}_k = \mathbf{Q}(\xi_k)$$

and for the non-axisymmetric function $\mathbf{q}(x, y)$, $\rho = \sqrt{\xi^2 + \eta^2}$ we have

$$B_k = \iint_\Omega \mathbf{q}(\xi, \eta) e^{i\zeta_k(\rho - r)} \sqrt{\frac{r}{\rho}}\, d\xi\, d\eta$$

From (3.2) we can obtain an asymptotic representation for the stress vector $\boldsymbol{\sigma}$ appearing in the expression (1.1) for the energy density, by taking into account the relations

$$\mathbf{u} = \sum_k \mathbf{u}_k + O(r^{-3/2}), \quad \mathbf{u}_k = \mathbf{a}_k e^{i\zeta_k r}/\sqrt{r}$$

$$\frac{\partial \mathbf{u}}{\partial r} = i\sum_k \zeta_k \mathbf{u}_k + O(r^{-3/2}), \quad \frac{\partial \mathbf{u}}{\partial z} = \sum_k \mathbf{u}_k' + O(r^{-3/2}), \quad r \to \infty$$

The derivatives $(\mathbf{u}_k')$ with respect to z of the vector $\mathbf{u}_k$ are expressed in terms of M', N', P', R', S', the latter representing the derivatives of the elements of the matrix K which are determined simultaneously with the functions M, N, P, R, S themselves. For the vector $\boldsymbol{\sigma} = \{\sigma_1, \sigma_2, \sigma_3\}$ we have

$$\boldsymbol{\sigma} = \sum_k (\lambda \mathbf{n} \operatorname{div} \mathbf{u}_k + 2\mu i\zeta_k \mathbf{u}_k + \mu(\mathbf{n} \times \operatorname{rot} \mathbf{u}_k)) + O(r^{-3/2}), \quad r \to \infty$$

$$\operatorname{div} \mathbf{u}_k = i\zeta_k (u_{k,1}\cos\varphi + u_{k,2}\sin\varphi) + u_{k,3}'$$

$$\operatorname{rot} \mathbf{u}_k = \mathbf{i}(i\zeta_k u_{k,3} - u_{k,2}') + \mathbf{j}(u_{k,1}' - i\zeta_k u_{k,3}) + \mathbf{k} i\zeta_k (u_{k,2} - u_{k,1}); \quad \mathbf{u}_k = \{u_{k,1}, u_{k,2}, u_{k,3}\}$$

The integrand function in (3.1) is of the order of unity when $r \to \infty$, since $\mathbf{u}, \boldsymbol{\sigma} \sim r^{-1/2}$ when $r \to \infty$, $z = \text{const}$. If S is a lower hemisphere of radius $R = \sqrt{x^2 + y^2 + z^2} \gg 1$, then ($\rho_V$ is the energy flux density across the hemisphere)

$$E_V = \int_0^{2\pi} \int_{\pi/2}^{\pi} \rho_V R^2 \sin\psi\, d\psi\, d\varphi \tag{3.3}$$

For $\psi \neq \pi/2$, $R \to \infty$ the asymptotic representation of $\mathbf{u}$ has the form

$$\mathbf{u} = \sum_{s=1}^{2} \mathbf{u}_s + O(R^{-2}), \quad R \to \infty, \quad \mathbf{u}_s = \mathbf{b}_s e^{iR\varkappa_s}/R$$

$$\mathbf{b}_s = \frac{i\cos\psi}{2\pi} K_s(\alpha_{1,s}, \alpha_{2,s}) \mathbf{Q}(\alpha_{1,s}, \alpha_{2,s}) \varkappa_s$$

$$\alpha_{1,s} = -\varkappa_s \sin\psi\cos\varphi, \quad \alpha_{2,s} = -\varkappa_s \sin\psi \sin\varphi, \quad s = 1, 2$$

We assume that when $z \to -\infty$, then the matrix K has the following representation:

$$K(\alpha_1, \alpha_2, z) \sim \sum_{s=1}^{2} K_s(\alpha_1, \alpha_2) e^{\sigma_s z}, \quad z \to -\infty$$

$$\sigma_s = \sqrt{\alpha^2 - \varkappa_s^2}, \quad \varkappa_1^2 = \lim_{z \to -\infty} \frac{\rho(z)\omega^2}{\lambda(z) + 2\mu(z)}, \quad \varkappa_2^2 = \lim_{z \to -\infty} \frac{\rho(z)\omega^2}{\mu(z)}$$

which is true, provided that λ, μ, ρ tends, as $z \to -\infty$, to constant values or increase, at most, according to a power law. We have

$$\boldsymbol{\sigma} = \sum_{s=1}^{2} \boldsymbol{\sigma}_s + O(R^{-2}), \quad R \to \infty$$

$$\boldsymbol{\sigma}_s = (\lambda \mathbf{n} d_s + 2i\mu\varkappa_s \mathbf{b}_s + \mu(\mathbf{n} \times \mathbf{r}_s)) e^{iR\varkappa_s}/R$$

$$d_s = i\varkappa_s [\sin\psi(b_{s,1}\cos\varphi + b_{s,2}\sin\varphi) + b_{s,3}\cos\psi]$$

$$\mathbf{r}_s = [\mathbf{i}(b_{s,3}\sin\varphi\sin\psi - b_{s,2}\cos\psi) + \mathbf{j}(b_{s,1}\cos\psi -$$

*) Here and henceforth we use the results of the paper given in the previous footnote.

$$b_{s,3}\cos\varphi\sin\psi) + \mathbf{k}\sin\psi\,(b_{s,2}\cos\varphi - b_{s,1}\sin\varphi)]\,i\varkappa_s$$
$$\mathbf{b}_s = \{b_{s,1},\ b_{s,2},\ b_{s,3}\}$$

Here $\mathbf{u}, \sigma \sim R^{-1}$, $R \to \infty$, therefore the integrand function in (3.3) is $\rho_V R^2 \sim 1$, $R \to \infty$. Thus, by virtue of the asymptotic representations obtained, ρ_V and ρ_R are the energy densities of the volume and Rayleigh waves, respectively. Since $\mathbf{u}_1$, $\mathbf{u}_2$ are the longitudinal and transverse waves respectively, if follows that $\rho_{V,p} = {}^1/_2\omega\,\mathrm{Im}\,(\sigma_1, \mathbf{u}_1)$ and $\rho_{V,s} = {}^1/_2\omega\,\mathrm{Im}\,(\sigma_2, \mathbf{u}_2)$ denote the energy

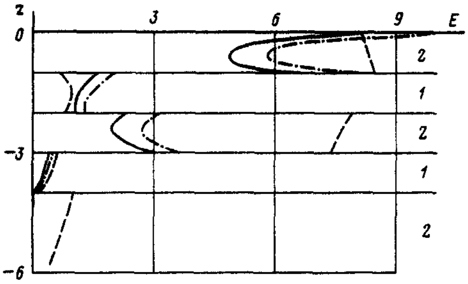

Fig.3

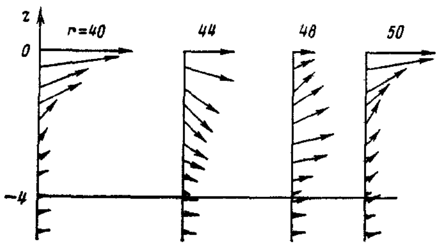

Fig.4

densities of the longitudinal and transverse waves, respectively. In an elastic inhomogeneous half-space the energy is transmitted, within the accuracy of up to the terms tending to zero as $R, r \to \infty$, across the side surface of the cylinder by the Rayleigh waves and through the surface of the lower hemisphere by the volume waves. Indeed, it can be shown that for the type of Rayleigh waves $\mathbf{u} \sim e^{-cR}$, $c > 0$ when $R \to \infty$, $\psi > \pi/2$ are valid, while for the volume waves we have $\mathbf{u} \sim r^{-3/2}$ when $r \to \infty$, $z = \mathrm{const}$. From this it follows that $\rho_V r \sim r^{-2}$; $r \to \infty$, $\rho_R R^2 \sim e^{-cR}$, $R \to \infty$.

The above computations show that the energy of the volume waves E_V calculated from (3.3) by integrating the energy density of the longitudinal and transverse waves over the surface of the lower hemisphere, coincides with the value of E_V obtained by integrating from zero to $\varkappa$ (formula (2.2)). Similarly, the energy of the Rayleigh waves E_R calculated from (3.1) by integrating the energy density of the Rayleigh waves over the side surface of the cylinder coincides with the value of E_R obtained as the sum of the **residues** (see (2.2)). In a zone situated at some distance from the source we can construct, with help of (3.1) and (3.3), the expressions for the energy density of the volume ρ_V and Rayleigh ρ_R waves not only in the direction determined by the normal to the surface under consideration, but also in the other two directions orthogonal to this normal and to each other. To do this it is sufficient to take the corresponding direction of the normal $\mathbf{n}$ in the expression for the energy density. The resulting three quantities represent the projections of the energy density vector $\boldsymbol{\rho}$ on the three directions chosen. The vector (Umov vector) determines the amount and direction of the energy transmitted through the given point of the medium.

Fig.3 depicts the dependence of the energy density of the Rayleigh waves ρ_R on the depth z for $r \gg 1$, in the same five-layer half-space as in Fig.2, $\omega = 0.5$, $\rho_{R,x}$ is the tangential source and $\rho_{R,z}$ is the vertical source. The solid lines correspond to $r\rho_{R,x} \times 5{\times}10^2$, $\varphi = 0$, the dashed lines to $r\rho_{R,x} \times 5{\cdot}10^2$, $\varphi = \pi/2$ and the dash-dot lines to $r\rho_{R,z} \times 10^2$. We see that the energy flux density is greater in the more rigid layers (medium 2) than in the softer layers (medium 1) and increases near the layer boundaries, while in a homogeneous half-space ρ_R decreases monotonously.

The results shown in Fig.3 are independent of r. This is due to the fact that at $\omega = 0.5$ the elements of the matrix K have only a single pole yielding a significant contribution. If the number of poles is greater than one, then the pattern of the energy density ρ_R distribution over z is different for different r, although the total amount of energy E_R passing across the side surface of the cylinder remains constant. Thus in Fig.4 the vectors $r\rho_R$ are constructed at various distances r for the tangential source and direction $\varphi = \pi/2$. The medium here is a two-layer half-space and $h = 4$, $\omega = 0.5$.

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